EUCLIDEAN GEOMETRY

NOVEMBER 2008

QUESTION 7

7.1 Complete the statements below by filling in the missing word(s) so that the statements are CORRECT:

7.1.1 The angle subtended by a chord at the centre of a circle is … . (1)

7.1.2 The angle between the tangent and a chord is … . (1)

7.1.3 The opposite angles of a cyclic quadrilateral are … . (1)

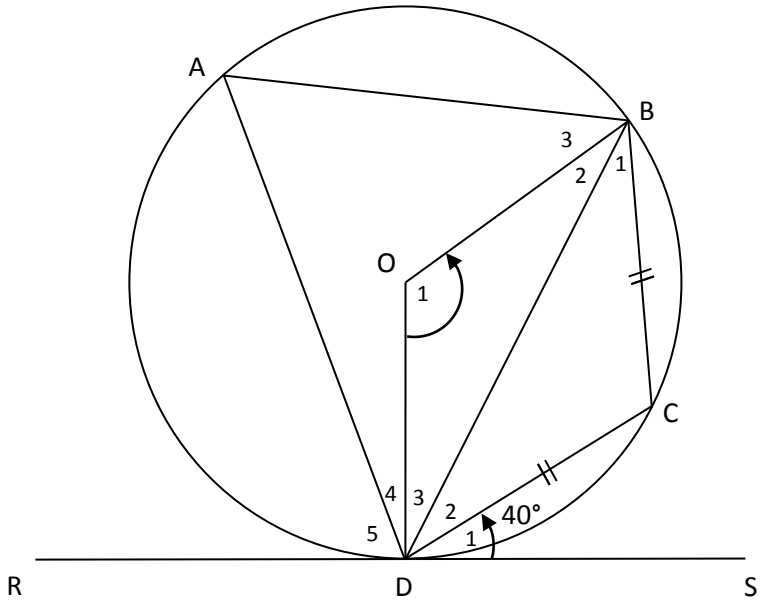
7.2 In the figure below, RDS is a tangent to circle O at D. If BC = DC and $C\hat{D}S = 40°$, calculate, with reasons, the measures of:

7.2.1 $B\hat{D}C$ (2)

7.2.2 $\hat{C}$ (1)

7.2.3 $\hat{A}$ (1)

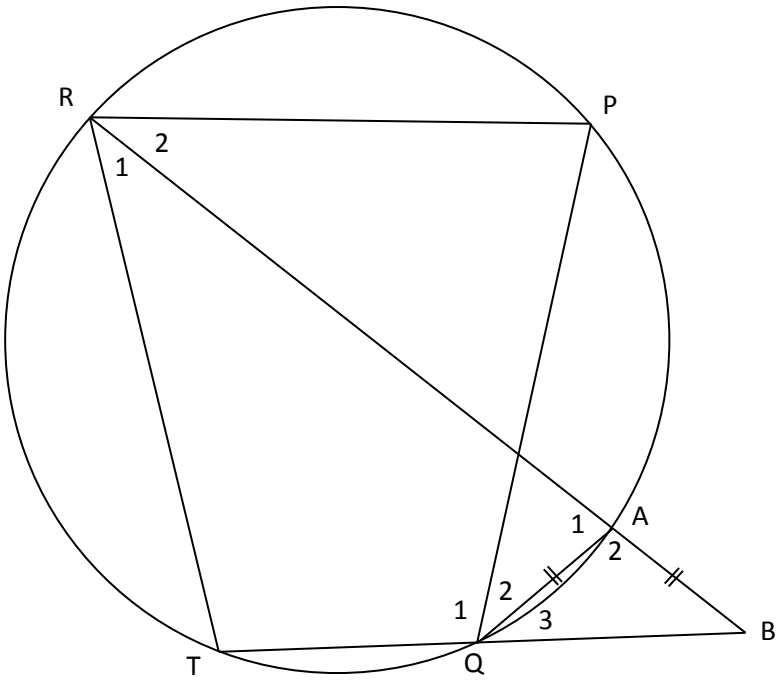
7.2.4 $\hat{O}_1$ (2)



[9]

QUESTION 8

In the diagram below, points R, P, A, Q and T lie on a circle. RA bisects $\hat{R}$ and AB = AQ. RA and TQ produced meet at B.



Prove that:

8.1 AQ bisects $P\hat{Q}B$ (3)

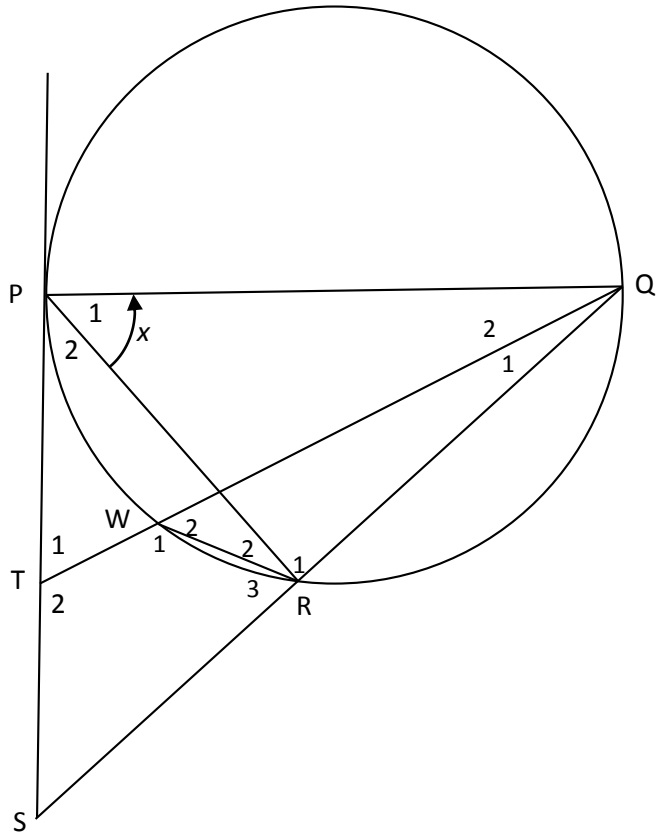
8.2 TR = TB (2)

8.3 $\hat{P} = T\hat{R}P$ (3)
[8]

QUESTION 9

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

Let $\hat{P}_1 = x$



9.1 Why is $P\hat{R}Q = 90°$? (1)

9.2 Prove that $\hat{P}_1 = \hat{S}$. (3)

9.3 Prove that SRWT is a cylic quadrilateral. (3)

9.4 Prove that $\Delta QWR \ /// \ \Delta QST$. (3)

9.5 If QW = 5 cm, TW = 3 cm, QR = 4 cm and WR = 2 cm, calculate the length of:

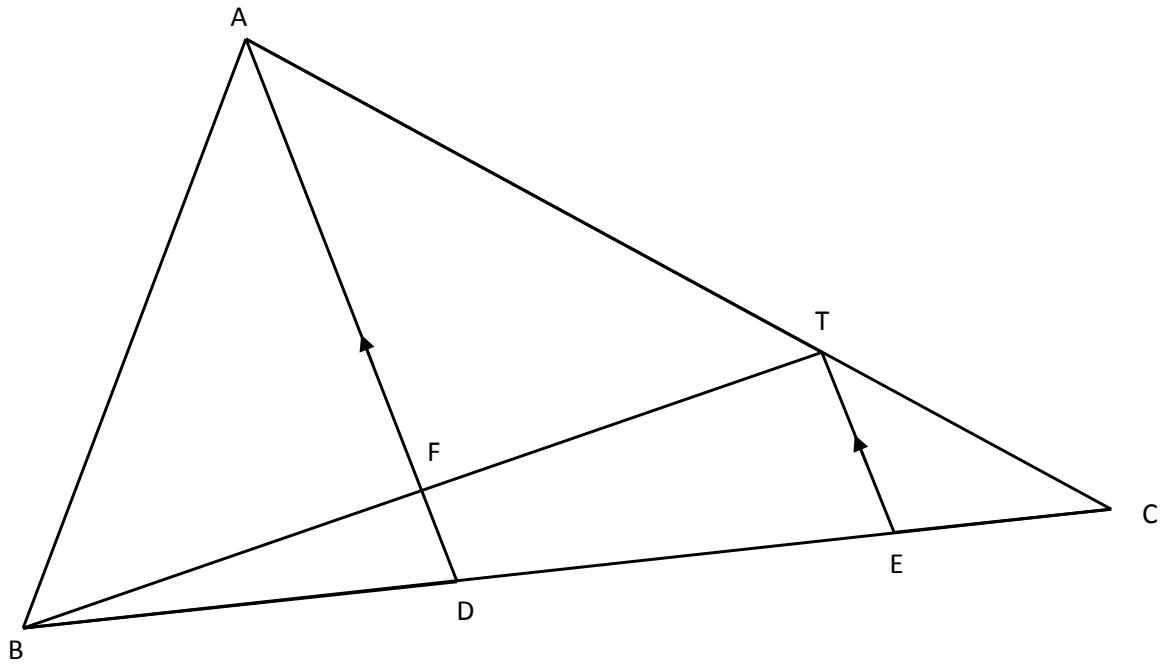
9.5.1 TS (3)

9.5.2 SR (3)

[16]

QUESTION 10

In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD || TE.



10.1 Write down the numerical value of $\frac{CE}{ED}$ (1)

10.2 Show that D is the midpoint of BE. (2)

10.3 If FD = 2 cm, calculate the length of TE. (2)

10.4 Calculate the numerical value of:

104.1 $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (1)

10.4.2 $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

[9]

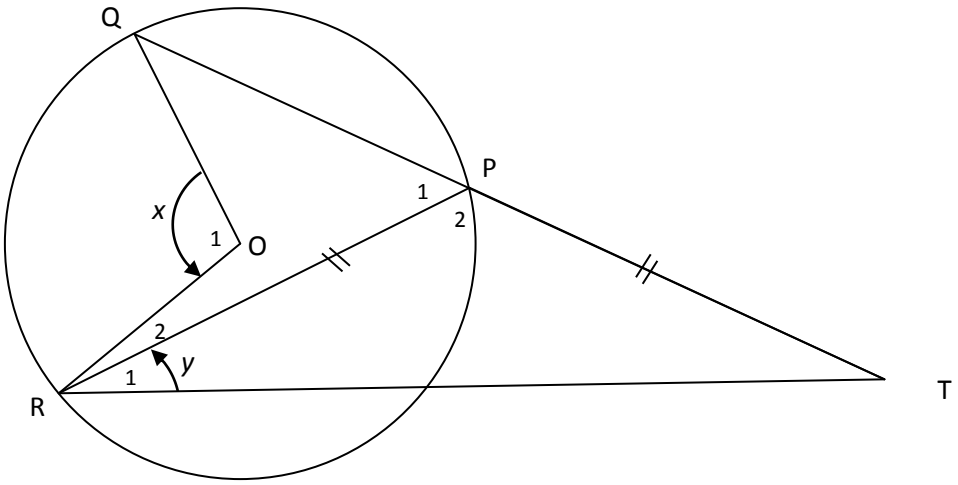
FEBRUARY / MARCH 2009

QUESTION 6

6.1 Complete the statement below by filling in the missing word(s) so that the statement is CORRECT:

The angle subtended by a chord or arc at the centre of a circle is … (1)

6.2 In the figure below, O is the centre of the circle and PT = PR.
Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$.



6.2.1 Express x in terms of y. (3)

6.2.2 If TQ = TR and $x = 120°$, calculate the measure of:

(a) y (2)

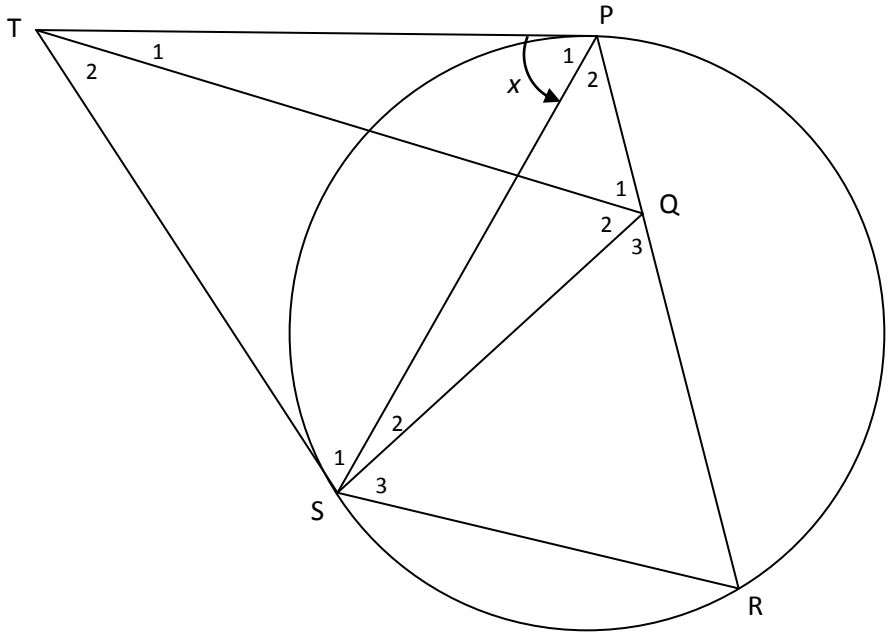
(b) $\hat{R}_2$ (Hint: Draw QR) (3)
[9]

QUESTION 7

In the figure TP and TS are tangents to the given circle. R is a point on the circumference.

Q is a point on PR such that $\hat{Q}_1 = \hat{P}_1$.
SQ is drawn.

Let $\hat{P}_1 = x$.



Prove that:

7.1 TQ || SR (4)

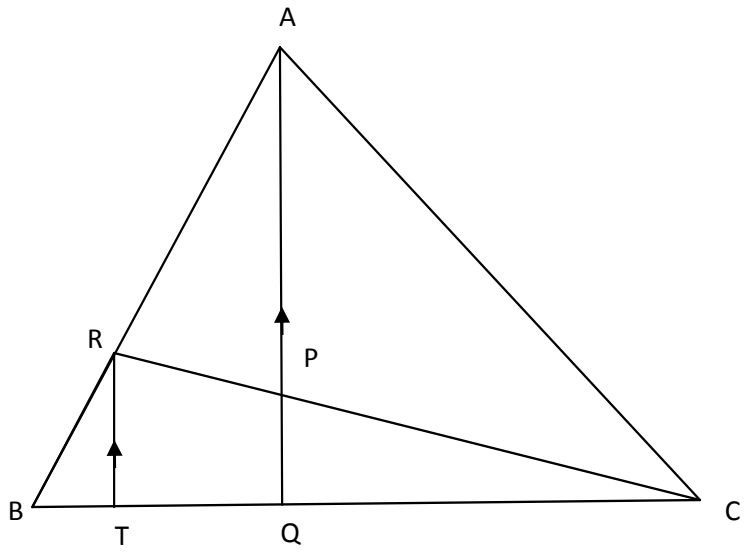
7.2 QPTS is a cyclic quadrilateral (4)

7.3 TQ bisects $S\hat{Q}P$ (3)

[11]

QUESTION 8

In the figure $AQ \parallel RT$, $\frac{BQ}{QC} = \frac{3}{5}$ and $\frac{BR}{RA} = \frac{1}{2}$.



8.1 If $BT = k$, calculate TQ in terms of k. (3)

8.2 Hence, or otherwise, calculate the numerical value of:

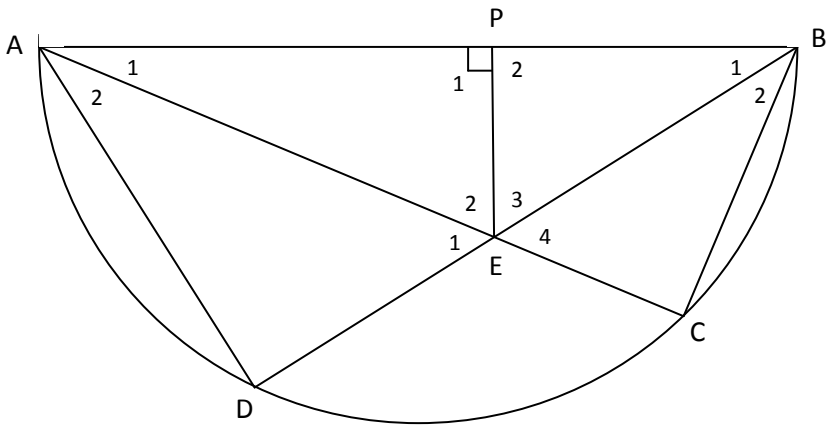
8.2.1 $\frac{CP}{PR}$ (3)

8.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC}$ (4)

[10]

QUESTION 9

In the accompanying figure, AB is the diameter of circle ADCB. Chords AC and BD intersect at E. EP is perpendicular to AB.



9.1 Prove that $\Delta BPE \;|||\; \Delta BDA$. (3)

9.2 Hence show that $\frac{BP}{BD} = \frac{PE}{AD}$. (2)

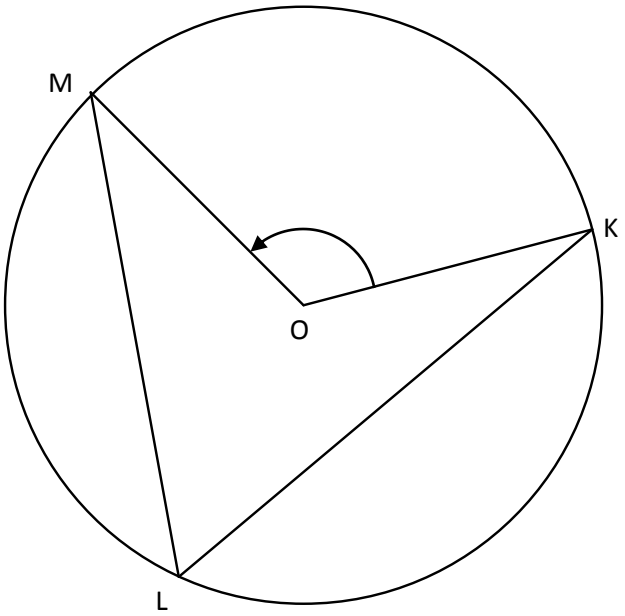
9.3 Prove that $AB^2 = BD^2 + \frac{BD^2.PE^2}{BP^2}$. (5)

[10]

NOVEMBER 2009

QUESTION 8

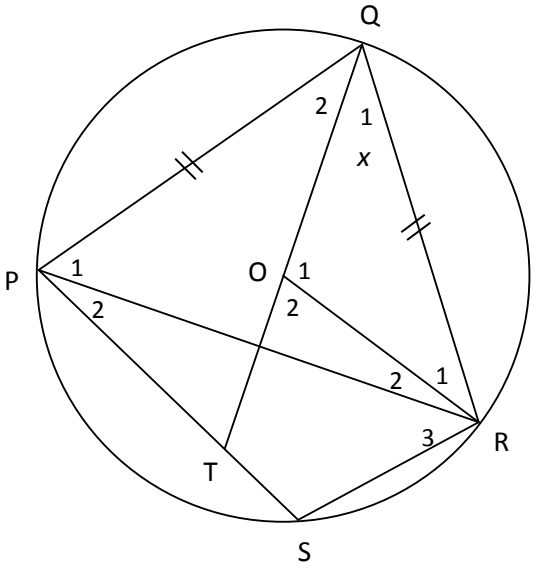
8.1 In the diagram below, O is the centre of the circle. K, L and M are points on the circumference of the circle.



Prove that the obtuse angle at O, $K\hat{O}M = 2K\hat{L}M$. (6)

8.2 In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS.

PQ = QR and $\hat{Q}_1 = x$.



8.2.1 Calculate, with reasons, $\hat{P}_1$ in terms of *x*. (3)

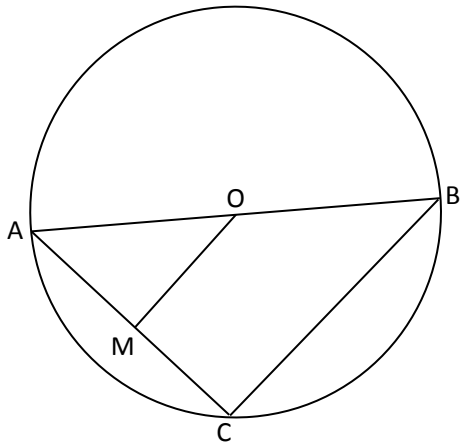
8.2.2 Show that TQ bisects $P\hat{Q}R$. (3)

8.2.3 Show that STOR is a cyclic quadrilateral. (3)

[15]

QUESTION 9

O is the centre of the circle below. OM ⊥ AC. The radius of the circle is equal to 5 cm and BC = 8 cm.



9.1 Write down the size of $B\hat{C}A$. (1)

9.2 Calculate:

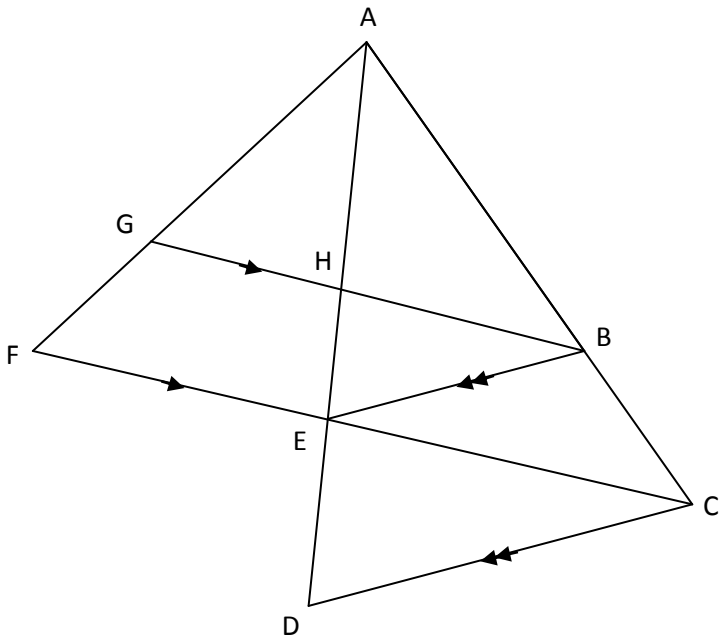
9.2.1 The length of AM, with reasons (3)

9.2.2 Area ΔAOM : Area ΔABC (3)

[7]

QUESTION 10

In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



10.1 Calculate with reasons:

10.1.1 AH : ED (4)

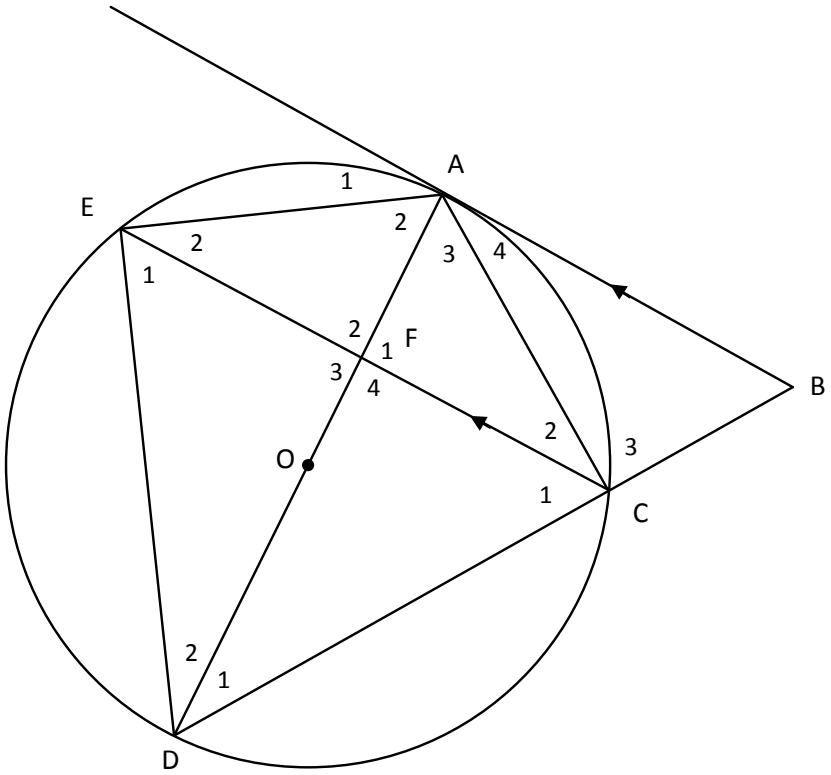
10.1.2 $\frac{BE}{CD}$ (2)

10.2 If HE = 2 cm, calculate the value of AD × HE. (2)

[8]

QUESTION 11

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



11.1 Show $\hat{C}_2 = \hat{D}_1$. (3)

11.2 Prove that ΔACF ||| ΔADC. (3)

11.3 Prove that AD = 4AF. (4)

[10]

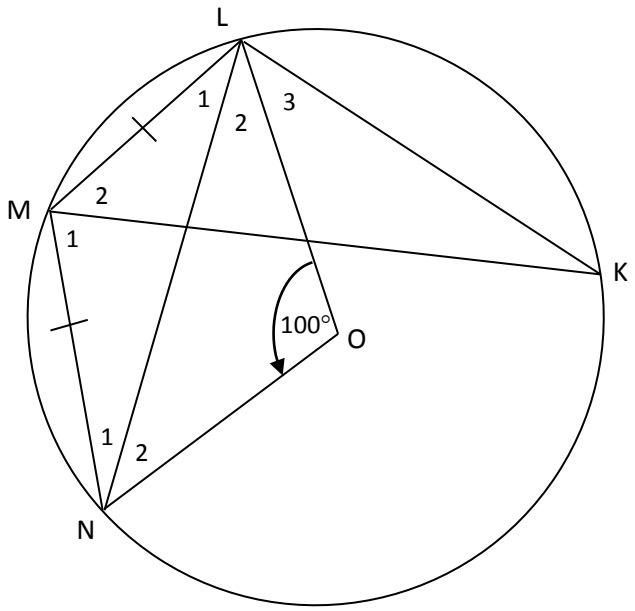
FEBRUARY / MARCH 2010

QUESTION 8

8.1 Complete the statement:

The sum of the angles around a point is ... (1)

8.2 In the figure below, O is the centre of the circle. K, L, M and N are points on the circumference of the circle such that LM = MN. $L\hat{O}N = 100°$.



Determine, with reasons, the values of the following:

8.2.1 $L\hat{M}N$ (3)

8.2.2 $L\hat{K}M$ (3)

[7]

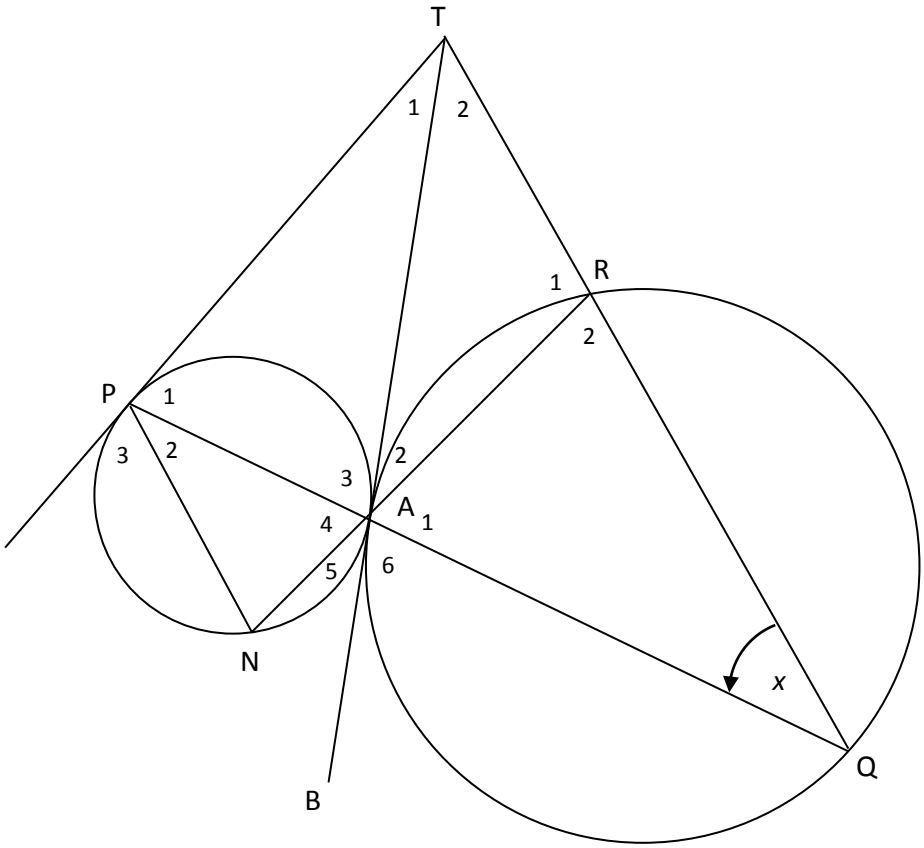
QUESTION 9

9.1 Complete the following statement:

The angle between the tangent and the chord … (1)

9.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines.

Let $\hat{Q} = x$.



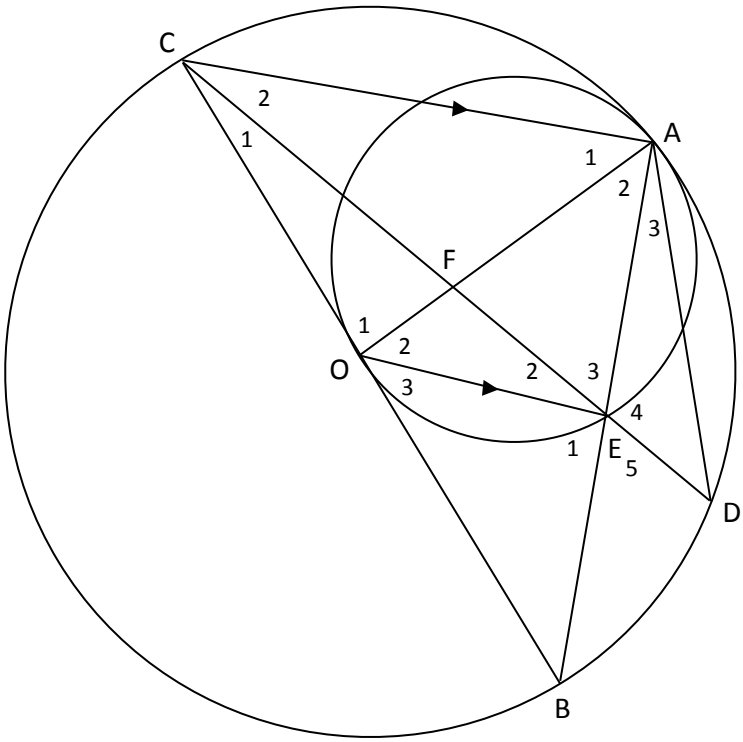
9.2.1 Name, with reasons, THREE other angles equal to *x*. (5)

9.2.2 Prove that APTR is a cyclic quadrilateral. (5)

[11]

QUESTION 10

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle.
OE || CA.



10.1 Prove, with reasons, that AE = BE. (2)

10.2 Prove that $\Delta AED \;|||\; \Delta CEB$. (3)

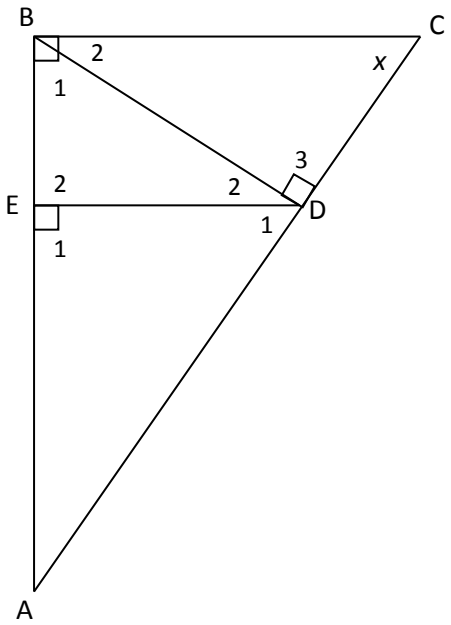
10.3 Hence, or otherwise, show that $AE^2 = DE.CE$. (2)

10.4 If $AE.EB = EF.EC$, show that E is the midpoint of DF. (3)
[10]

QUESTION 11

ΔABC is a right-angled triangle with $\hat{B} = 90°$. D is a point on AC such that BD $\perp$ AC and E is a point on AB such that DE $\perp$ AB. E and D are joined.

AD : DC = 3 : 2.
AD = 15 cm.



11.1 Prove that $\Delta BDA \,|||\, \Delta CDB$. (3)

11.2 Calculate BD (Leave your answer in surd form). (3)

11.3 Calculate AE (Leave your answer in surd form). (6)
[12]

NOVEMBER 2010

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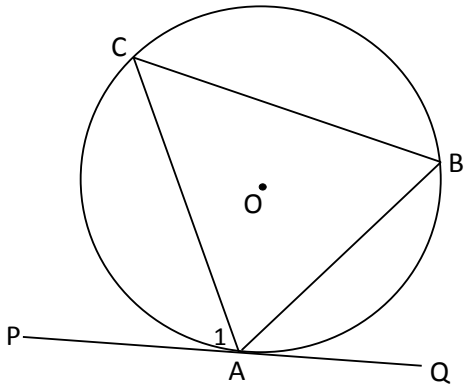
NOVEMBER 2011

QUESTION 8

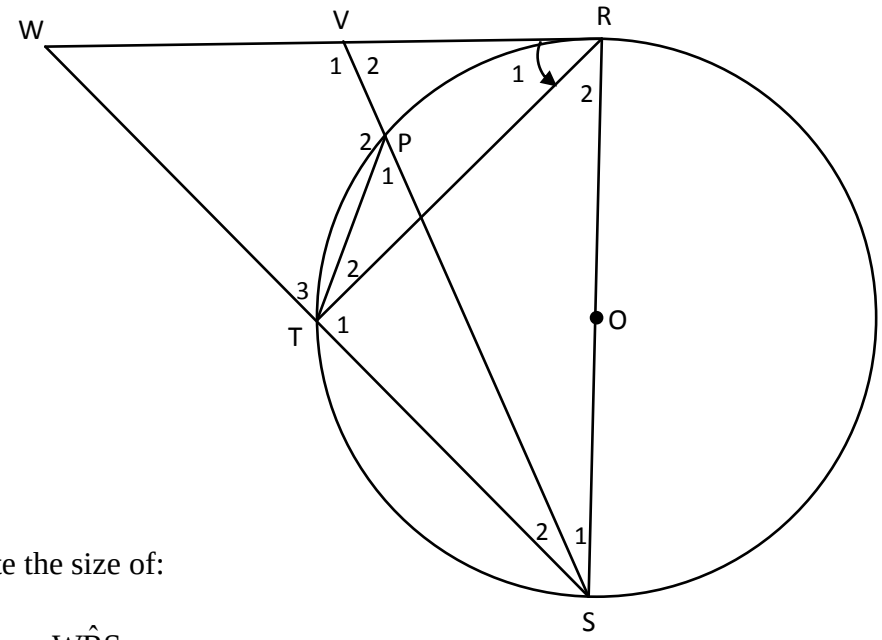
8.1 In the diagram below is O the centre of the circle. PQ is a tangent to the circle at A. B and C are points on the circumference of the circle. AB, AC and BC are joint.

Prove the theorem that allege that $C\hat{A}P = A\hat{B}C$.



(5)

8.2 RS is a diameter of the circle with centre O. Chord ST is extend to W. Chord SP is lengthen, meet the tangent RW at V. $\hat{R}_1 = 50°$.



Calculate the size of:

8.2.1 $W\hat{R}S$ (1)

8.2.2 $\hat{W}$ (2)

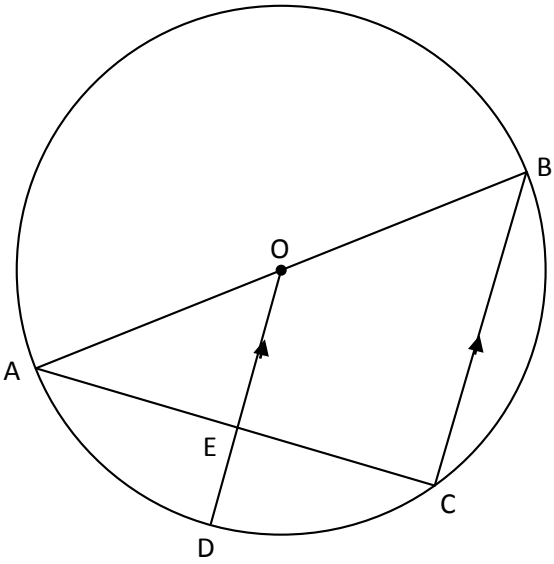
8.2.3 $\hat{P}_1$ (3)

8.2.4 Prove that $\hat{V}_1 = P\hat{T}S$. (4)

[15]

QUESTION 9

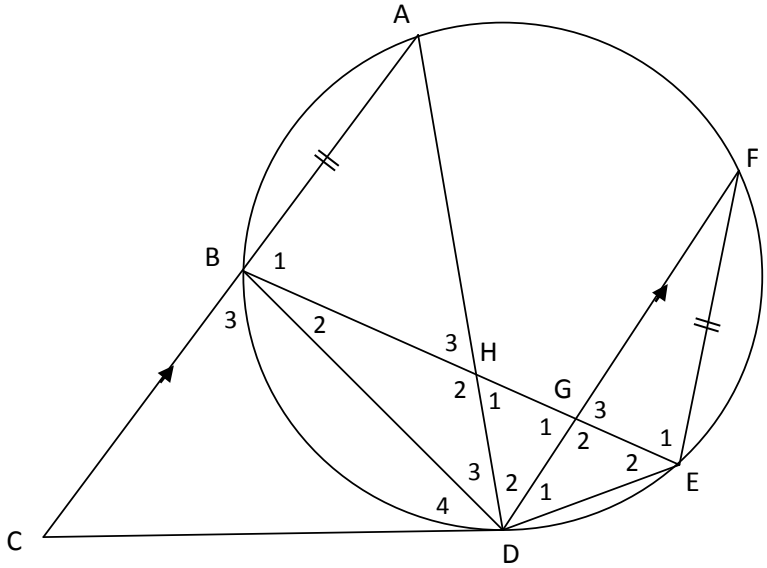
AB is a diameter of the circle ABCD. OD is parallel to BC and meet AC in E.



If the radius is 10 cm and AC = 16 cm, calculate the length of ED. **[5]**

QUESTION 10

CD is a tangent to the circle ABDEF at D. Chord AB is extended to C. Chord BE cuts chord AD in H and chord FD in G. AC || FD and FE = AB. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



10.1 Determine THREE other angles equal to x. (6)

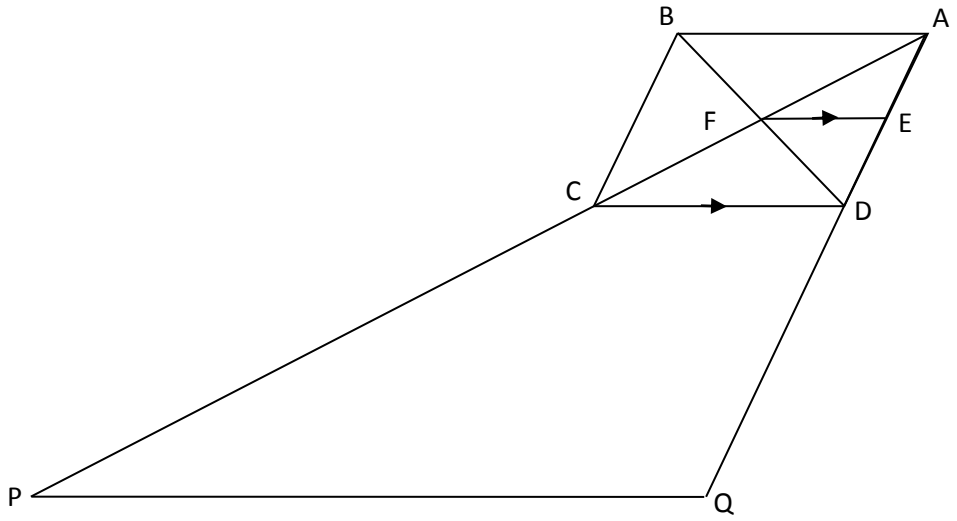
10.2 Prove that $\Delta BHD \,|||\, \Delta FED$. (5)

10.3 Hence, or otherwise, prove that AB.BD = FD.BH. (2)

[13]

QUESTION 11

ABCD is a parallelogram with diagonals that cut in F. FE is parallel to CD extended.
AC is extended to P so that PC = 2AC and AD is extended to Q so that DQ = 2AD.



11.1 Show that E is the centre of AD. (2)

11.2 Prove that PQ || FE. (3)

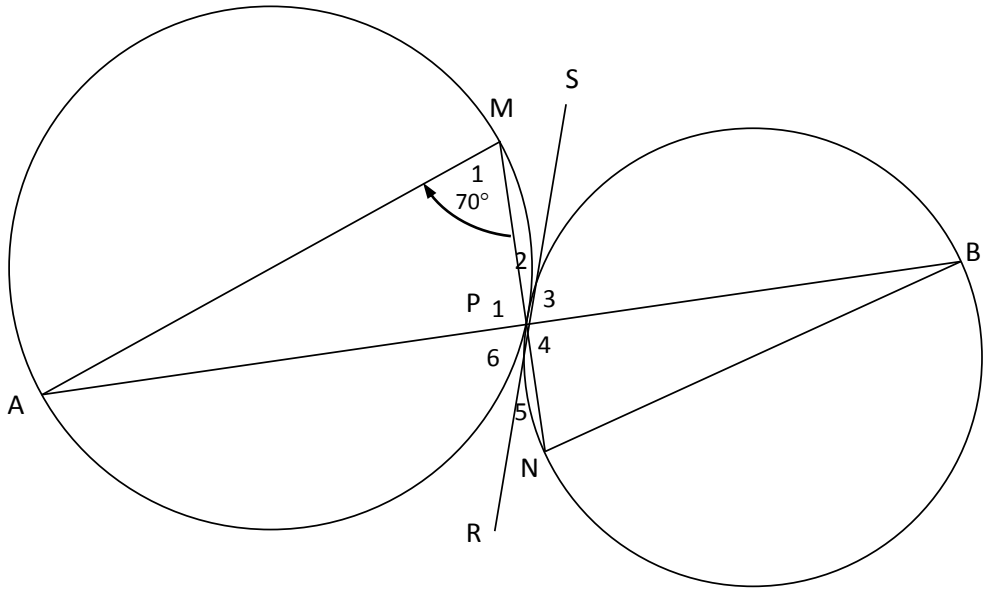
11.3 If PQ is equal to 60 cm, calculate the length of FE. (5)
[10]

FEBRUARY / MARCH 2012

In the next FOUR questions, ensure you give reasons for EACH statement you make.

QUESTION 8

In the diagram below, AM is the diameter of the bigger circle AMP. RPS is a common tangent to both circles at P. APB and MPN are straight lines.



8.1 State the size of $\hat{P}_1$. (1)

8.2 Hence, show that BN is the diameter of the smaller circle. (2)

8.3 If $\hat{M}_1 = 70°$, calculate the size of each of the following angles:

8.3.1 $\hat{A}$ (1)

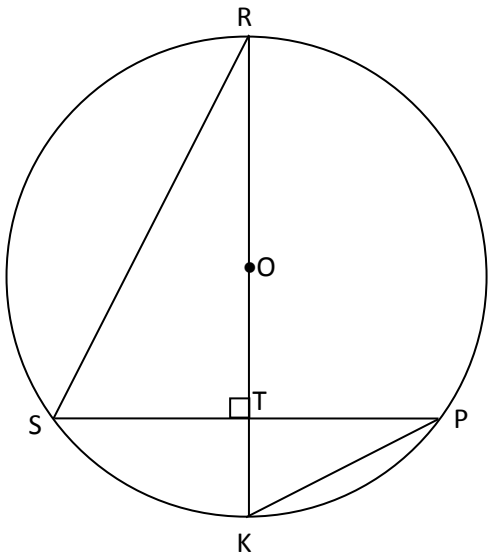
8.3.2 $\hat{P}_6$ (1)

8.3.3 $\hat{B}$ (2)

[7]

QUESTION 9

In the diagram below, O is the centre of the circle with diameter RK.
PS ⊥ RK.
RK intersects PS at T.



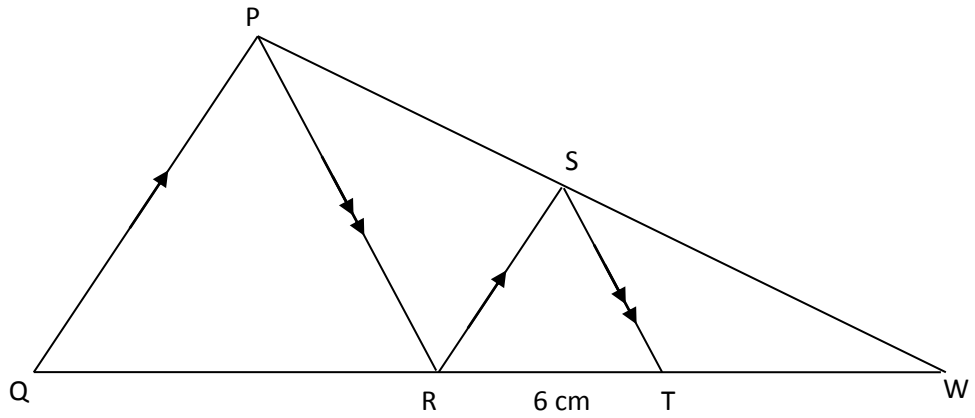
9.1 If PS = 4*x*, write down the length of ST in terms of *x*. (1)

9.2 Prove that ΔRST ||| ΔPKT. (3)

9.3 If it is further given that TK = *x* and RT = 320 mm, calculate the value of *x*. (3)
[7]

QUESTION 10

In ΔPQW, S is a point on PW and R is a point on QW such that SR || PQ.
T is a point on QW such that ST || PR.
RT = 6 cm
WS : SP = 3 : 2



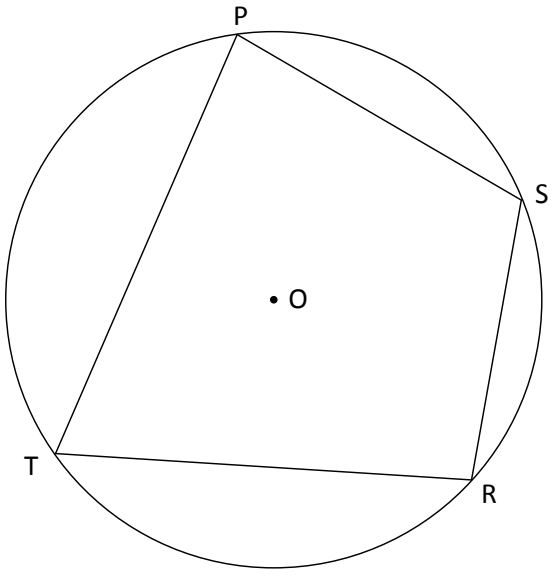
Calculate:

10.1 WT (3)

10.2 WQ (4)
[7]

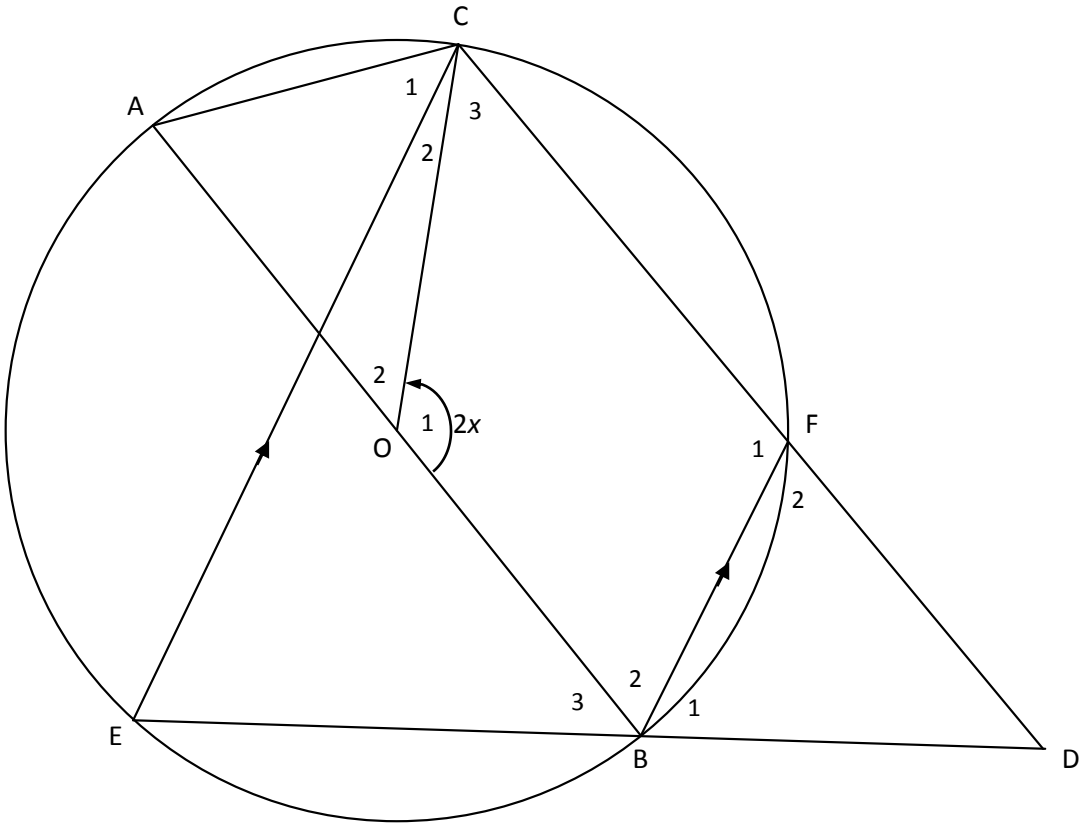
QUESTION 11

11.1 In the diagram below, O is the centre of the circle. PSRT is a cyclic quadrilateral.
Prove the theorem that states $P\hat{T}R + P\hat{S}R = 180°$.



(6)

11.2 In the diagram below, O is the centre of the circle. AB is a diameter of the circle. Chord CF produced meets chord EB produced at D. Chord EC is parallel to chord BF.
CO and AC are joined.
Let $\hat{O}_1 = 2x$



11.2.1 Determine, in terms of x, the size of $\hat{F}_1$. (4)

11.2.2 Prove that DF = BD. (4)

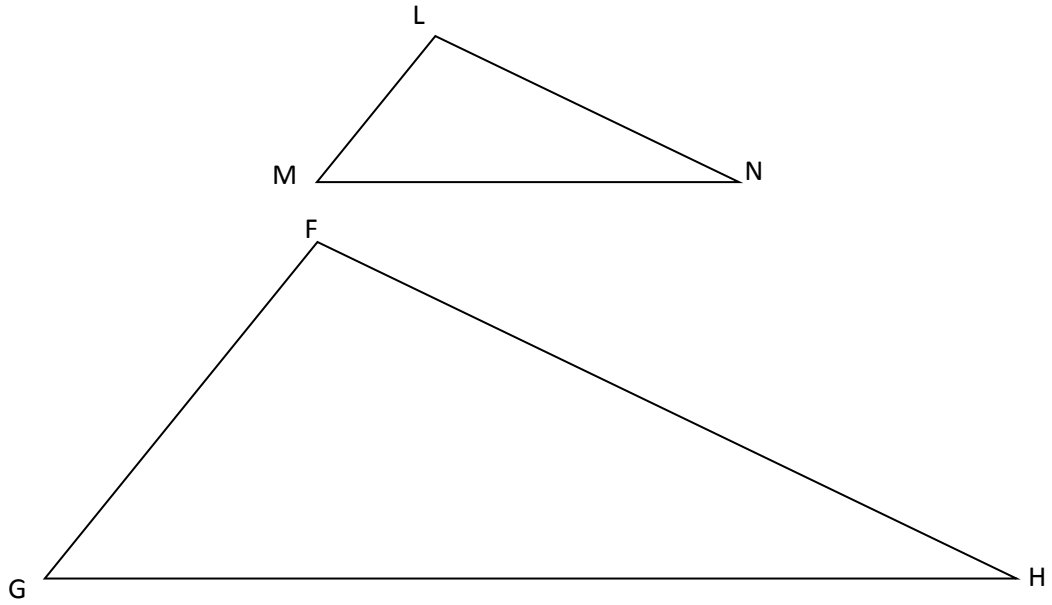
11.2.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)

11.2.4 If DF = 5 cm and OA = 6 cm, calculate area ΔBFD : area ΔAOC. (4)
[22]

NOVEMBER 2012

QUESTION 7

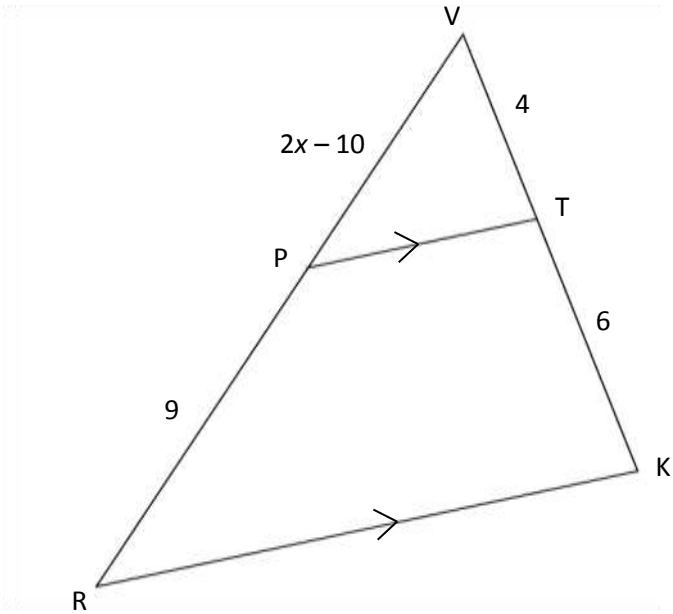
7.1 If in Δ LMN and Δ FGH is given that $\hat{L} = \hat{F}$ and $\hat{M} = \hat{G}$, prove the theorem that states that $\frac{LM}{FG} = \frac{LN}{FH}$.



(7)

7.2 In ΔVRK below, P lies on VR and T on VK so that PT || RK. VT = 4 units, PR = 9 units, TK = 6 units and VP = 2*x* – 10 units.

Calculate the value of *x*.



(4)
[11]

QUESTION 8

8.1 Complete the following theorem:

The angle between the tangent and the chord is equal to ... (1)

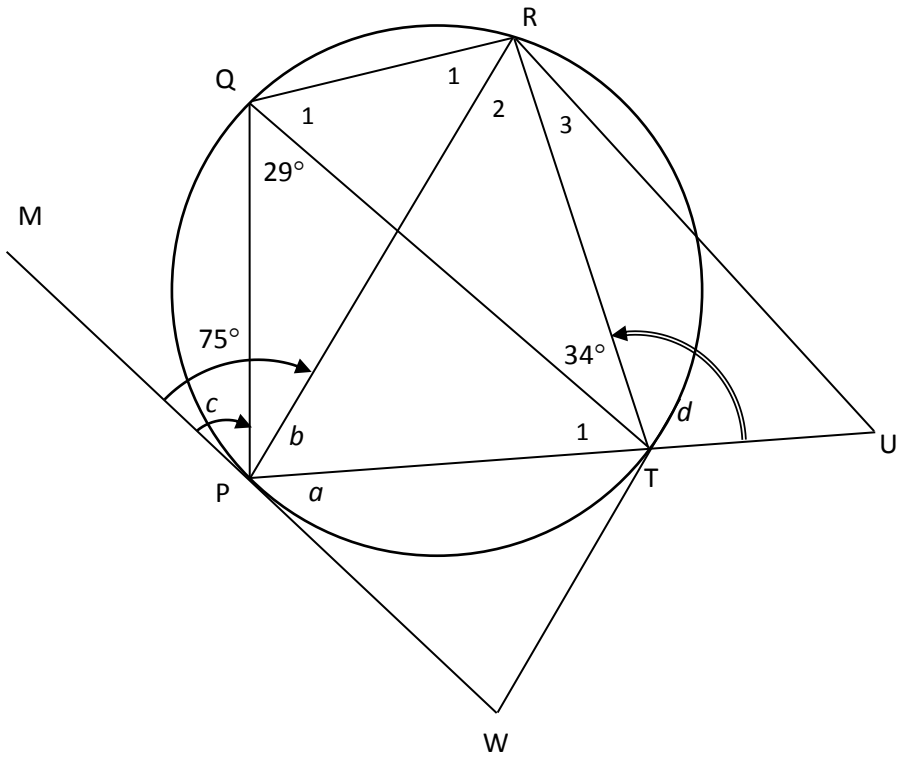
8.2 In the diagram points P, Q, R and T lie on the circumference of the circle. MW and TW are tangents to the circle at P and T respectively. PT is extended so that RU meets at U.

$M\hat{P}R = 75°$

$P\hat{Q}T = 29°$

$Q\hat{T}R = 34°$

Let $T\hat{P}W = a$, $R\hat{P}T = b$, $M\hat{P}Q = c$ and $R\hat{T}U = d$, calculate the value of *a*, *b*, *c* and *d*.

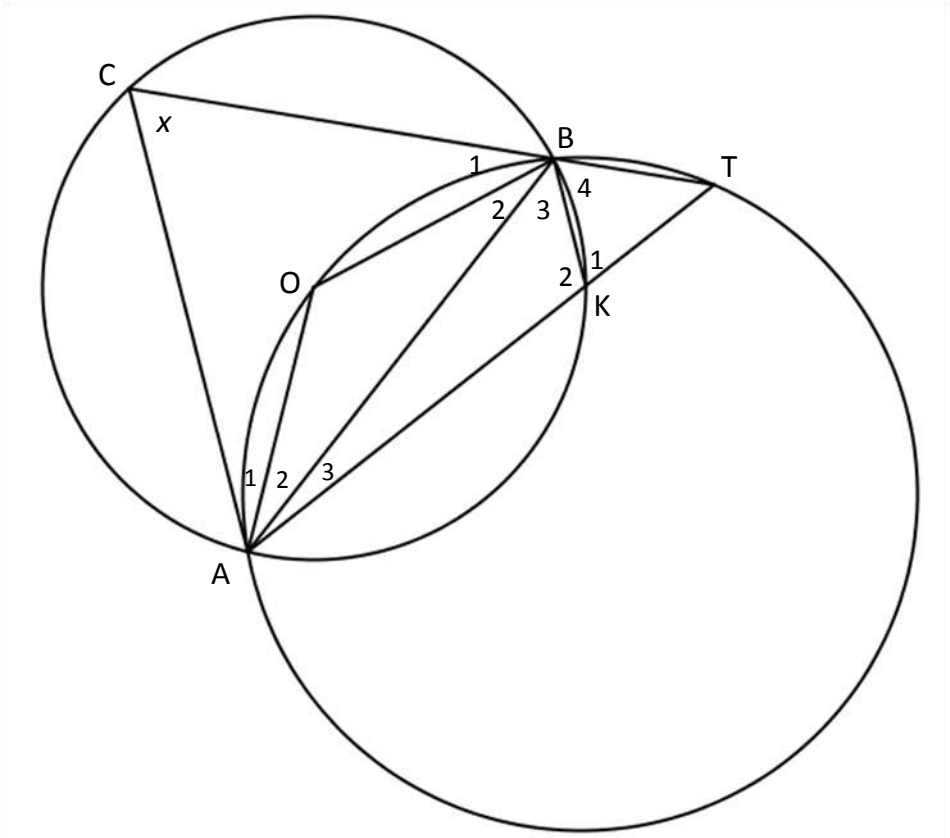


(9)
[10]

QUESTION 9

O is the centre of the circle CAKB. AK extended, cuts circle AOBT at T.

$A\hat{C}B = x$



9.1 Prove that $\hat{T} = 180° - 2x$. (3)

9.2 Prove AC || KB. (5)

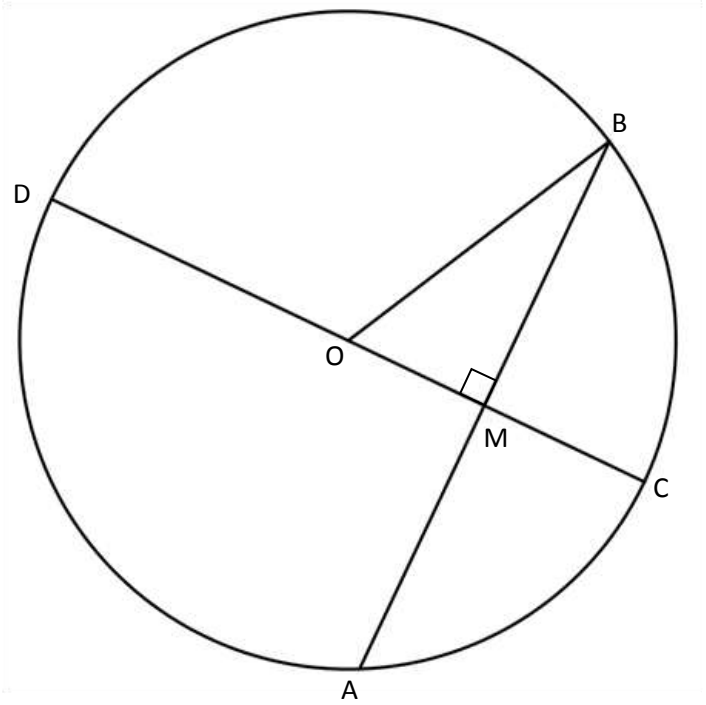
9.3 Prove $\Delta BKT \,|||\, \Delta CAT$. (3)

9.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$. (3)

[14]

QUESTION 10

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. CM : MD = 4 : 9 and AB = 24 units.



10.1 Determine an expression for DC in terms of x if CM = $4x$ units. (1)

10.2 Determine an expression for OM in terms of x. (2)

10.3 Hence, or otherwise, calculate the length of the radius. (4)

[7]

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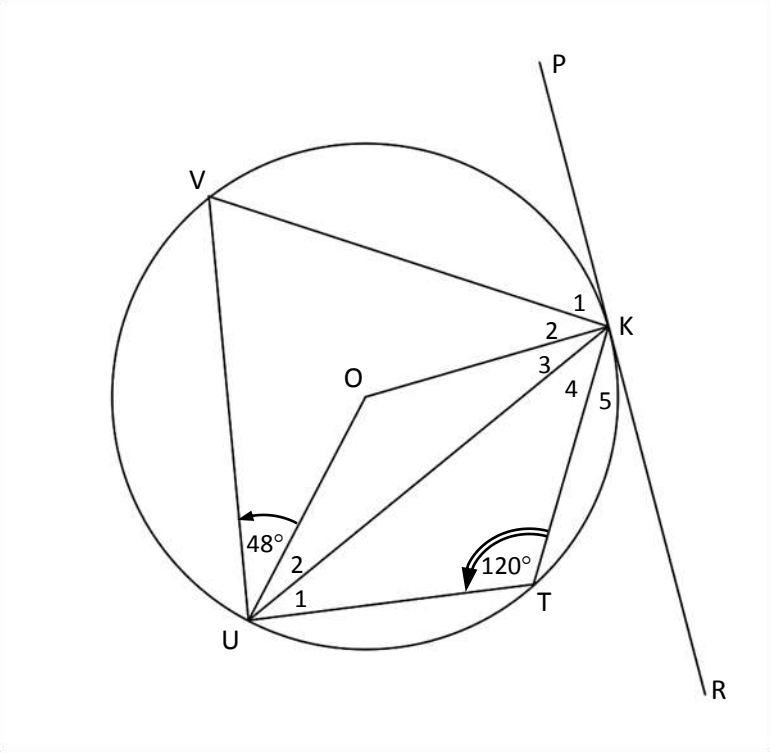
QUESTION 8

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K. $O\hat{U}V = 48°$ and $K\hat{T}U = 120°$.



Calculate, with reasons, the sizes of the following angles:

8.1 $\hat{V}$ (2)

8.2 $K\hat{O}U$ (2)

8.3 $\hat{U}_2$ (2)

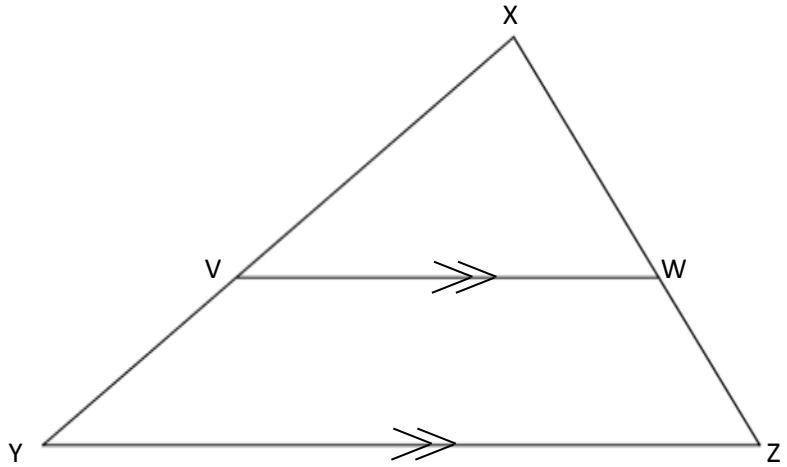
8.4 $\hat{K}_1$ (2)

8.5 $\hat{K}_2$ (2)

[10]

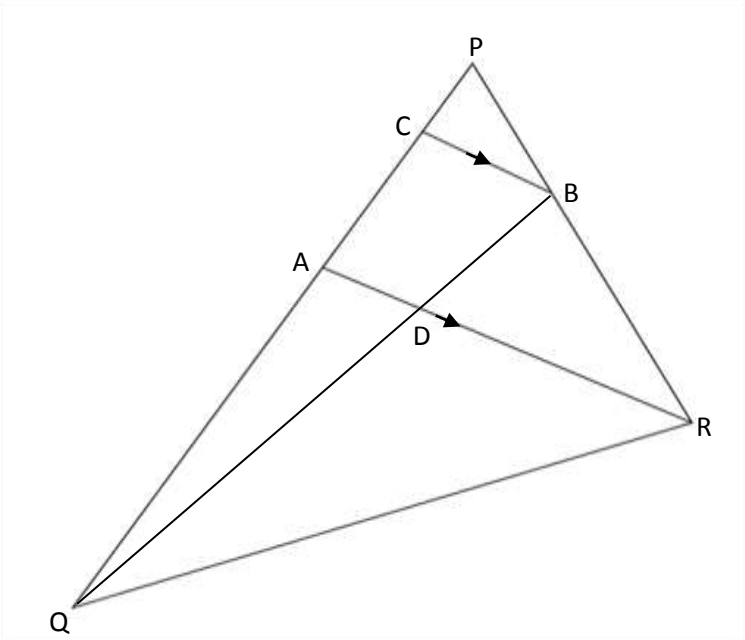
QUESTION 9

9.1 Use the diagram below to prove the theorem which states that if VW || YZ then $\frac{XV}{VY} = \frac{XW}{WZ}$.



(6)

9.2 In ΔPQR below, B lies on PR such that 2PB = BR. A lies on PQ such that PA : PQ = 3 : 8.
BC is drawn parallel to AR.



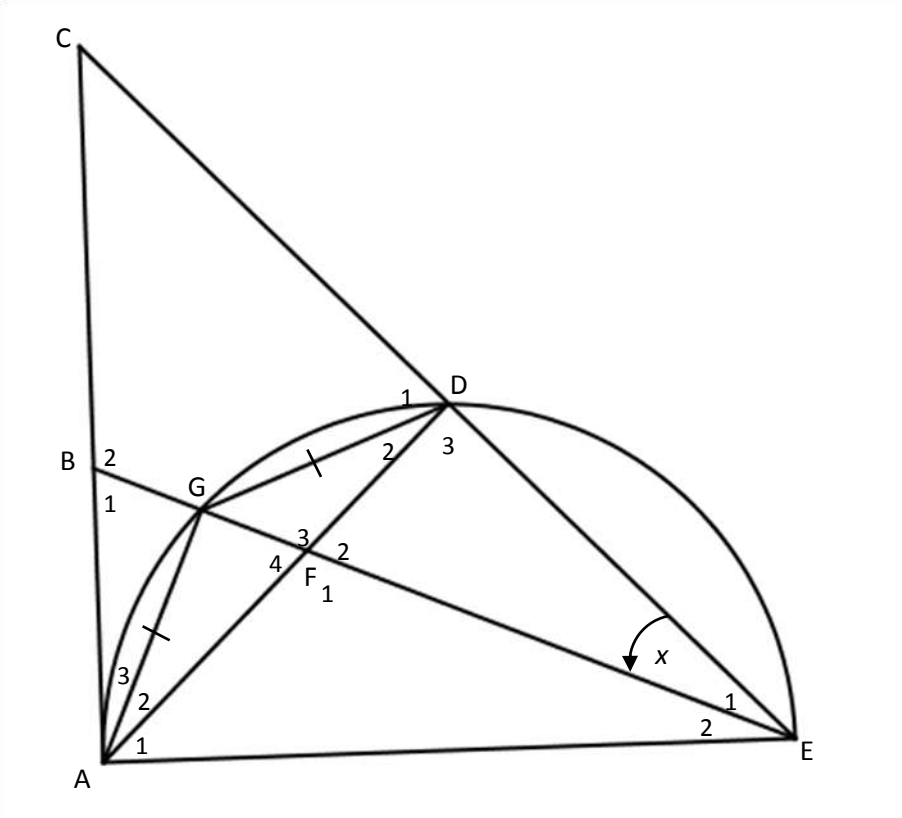
9.2.1 Write down the value of $\frac{\text{area of } \Delta PRA}{\text{area of } \Delta QRA}$ (2)

9.2.2 Calculate the value of the ratio $\frac{BD}{BQ}$. Show all working to support your answer. (5)

[13]

QUESTION 10

In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F.
AG = GD. $\hat{E}_1 = x$.



10.1 Write down, with reasons, FOUR other angles each equal to *x*. (8)

10.2 Prove that BE.DE = AE.FE (7)

10.3 Prove that $\hat{B}_1 = \hat{D}_1$ (4)

[19]